Quiz 8 Solution

February 23, 2018

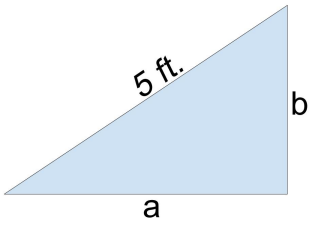
1. (3 points) The top of a 5-foot ladder is sliding down a wall at a rate of 2 ft/sec.

   (a) Sketch a picture and label it.

   (b) How quickly is the base of the ladder sliding away from the wall when the base of the ladder is 4 feet from the wall?

   Solution:

   (a)



   (b) We want $\frac{da}{dt}$ when $a = 4$ and $\frac{db}{dt} = -2$.
   We use the Pythagorean Theorem: $a^2+b^2 = 5^2$.
   Taking the derivative, we get $2a\frac{da}{dt}+2b\frac{db}{dt} = 0$.
   When $a = 4$, $b = \sqrt{5^2 - 4^2} = 3$.
   Substituting, we get $2(4)\frac{da}{dt} + 2(3)(-2) = 0$.
   Solving for $\frac{da}{dt}$, we get $\frac{da}{dt} = \frac{6}{4} = \frac{3}{2}$.

   Answer: The base of the ladder is sliding away from the wall at a rate of $\frac{3}{2}$ ft/sec.

2. (1 point) Which of the following is a critical number of $y = \frac{x-2}{x^2-4}$?

   (a) $x = -2$ (b) $x = 0$ (c) $x = 1$ (d) $x = 2$ (e) y has no critical numbers.

   Solution: Remember that a critical number is a number c where $f'(c) = 0$ or is undefined, and $f(c)$ exists.

   We first take the derivative:

$$\begin{aligned} y' &= \frac{(x^2-4)(1)-(x-2)(2x)}{(x^2-4)^2} \text{ by Quotient Rule.} \\ &= \frac{x^2-4-2x^2+4x}{(x^2-4)^2} \\ &= \frac{-x^2+4x-4}{(x^2-4)^2} \end{aligned}$$

So $y' = 0$ when $-x^2 + 4x - 4 = 0$ and is undefined when $(x^2 - 4)^2 = 0$.

$$\begin{aligned} -(x^2 - 4x + 4) &= 0 \\ x^2 - 4x + 4 &= 0 \\ (x-2)^2 &= 0 \\ x &= 2 \end{aligned} \qquad \begin{aligned} x^2 - 4 &= 0 \\ (x+2)(x-2) &= 0 \\ x &= -2, x = 2 \end{aligned}$$

However, y is undefined at both $x = 2$ and $x = -2$. So y has no critical values.

Answer: (e)

3. (1 point) What would you like to review on Wednesday?

   Answer: Answers will vary.